HRTFs can be calculated

Wave equation: $\frac{\partial^2 p'}{\partial t^2} = c^2\left(\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} + \frac{\partial^2 p'}{\partial z^2}\right) = c^2\nabla^2 p'$

Fourier Transform from Time to Frequency Domain: $p'(x, y, z, t) = \int_{-\infty}^{\infty} P(x, y, z, \omega) e^{-i\omega t} d\omega$

Helmholtz equation: $\nabla^2 P + k^2 P = 0$

Boundary conditions:

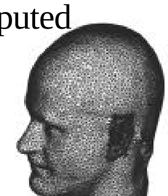
Sound-hard boundaries: $\frac{\partial P}{\partial n} = 0$

Sound-soft boundaries: $P = 0$

Impedance boundary conditions: $\frac{\partial P}{\partial n} + i\sigma P = g$

Sommerfeld radiation condition (for infinite domains): $\lim_{r \to \infty} r\left(\frac{\partial P}{\partial r} - ikP\right) = 0$

HRTFs can be computed



- Boundary Element Method
- Obtain a mesh
- Using Green's function *G*

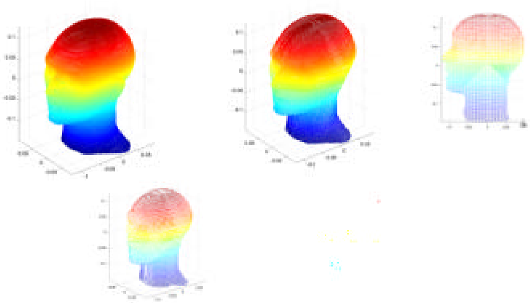
$$G(\mathbf{x},\mathbf{y}) = \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|}$$

- Convert equation and b.c.s to an integral equation

$$C(x)p(x) = \int_{\Gamma_y}\left[G(x,y;k)\frac{\partial p(y)}{\partial n_y} - \frac{\partial G(x,y;k)}{\partial n_y}p(y)\right] d\Gamma_y$$

- Need accurate surface meshes of individuals
- Obtain these via computer vision

Current work: Develop Meshes



Original Kemar surface points from Dr. Yuvi Kahana, ISVR, Southampton, UK

New quadric metric for simplifying meshes with appearance attributes

Hugues Hoppe
Microsoft Research

Presented by Zhihui Tang

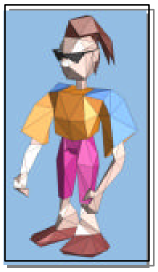
Introduction

- Several techniques have been developed for geometrically simplify them. Relatively few techniques account for appearance attributes during simplification.
- Metric introduced by Garland and Hecbert is fast and reasonably accurate. They can deal with appearance attribute.
- In this paper, developed an improved quadric error metric for simplifying meshes with attributes.

Advantage of the new metric:

- intuitively measures error by geometric correspondence
- less storage (linear on no. of attributes)
- evaluate fast (sparse quadric matrix)
- more accurate simplifications(experiments)

What is Triangle Meshes



Vertex 1	$x_1\ y_1\ z_1$	Face 1	1	2	3
Vertex 2	$x_2\ y_2\ z_2$	Face 2	1	2	4
Vertex 3	$x_3\ y_3\ z_3$	Face 3	2	4	5
……		……			

- Geometry $\mathbf{p}$? R^3
- attributes
  normals, colors, texture coords, ...

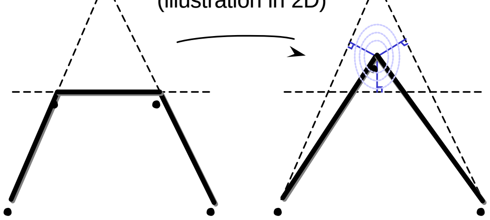
Notation

- A triangle mesh M is described by:
  V , F.
- Each vertex v in V has a geometric position p_v in $\mathbf{R}^3$ and A set of m attribute scalars s_v in $\mathbf{R}^m$. That is v is in $\mathbf{R}^{m+3}$.

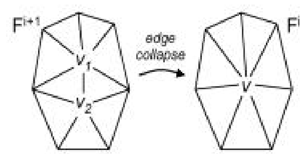
Previous Quadratic Error Metrics

- Minimize sum of squared distances to planes

(illustration in 2D)



Mesh simplification



Simplification of Geometry

$$Q^v(v) = \sum_{f \ni v} \text{area}(f) \cdot Q^f(v)$$

$Q^v(v) = Q^{v1}(v)+Q^{v2}(v)$

$Q^f(v=(p))=(n^t v+d)^2=v^t(nn^t)v+2dn^t v+d^2$

$=(A,b,c)=((nn^t),(dn),d^2)$

Q^f is stored using 10 coefficients.

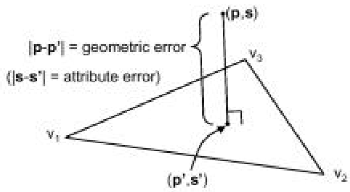
Vertex position v_{min} minimizing $Q^v(v)$ is the solution of $Av = -b$

Simplification of Geometry and Attributes

- This approach is to generalize the distances-to-plane metric in R^3 to a distance-to-hyperplane in R^{3+m}.
- $Q^f(v)=\|v-v'\|^2 =\|p-p'\|^2+\|s-s'\|^2$
- Storage requires (4+m)(5+m)/2 coefficients

New Quadric Error Metric

$$Q^f(\mathbf{v}=(\begin{smallmatrix}\mathbf{p}\\ \mathbf{s}\end{smallmatrix})) = Q^f_p(\mathbf{v}) + \sum_{j=1}^{m} Q^f_{s_j}(\mathbf{v})$$



New Quadric Error Metric

- $Q^f(v)=\|p-p'\|^2+\|s-s'\|^2$

$$Q^f_p = (\mathbf{A}, \mathbf{b}, c) = \left(\left(\begin{array}{c|c} \mathbf{n}\mathbf{n}^T & \ddots 0 \ddots \\ \hline \ddots 0 \ddots & \ddots 0 \ddots \end{array} \right), \left(\begin{array}{c} d\,\mathbf{n} \\ \hline 0 \end{array} \right), d^2 \right)$$

$$\hat{s}_j(\mathbf{p}) = \mathbf{g}_j^T\,\mathbf{p} + d_j$$

$$\begin{pmatrix} \mathbf{p}_1^T & 1 \\ \mathbf{p}_2^T & 1 \\ \mathbf{p}_3^T & 1 \\ \mathbf{n}^T & 0 \end{pmatrix} \begin{pmatrix} \mathbf{g}_j \\ \\ d_j \end{pmatrix} = \begin{pmatrix} s_{1,j} \\ s_{2,j} \\ s_{3,j} \\ 0 \end{pmatrix}$$

$$Q^f_{s_j}(\mathbf{v}) = (\hat{s}_j(\mathbf{p}) - s_j)^2 = (\mathbf{g}_j^T\mathbf{p} + d_j - s_j)$$

$Q^f_{s_j}(\mathbf{v})$ = **(A,b,**c) =

$$\left(\left(\begin{array}{c|ccc} \mathbf{g}_j\mathbf{g}_j^T & \ddots 0 \ddots & -\mathbf{g}_j & \ddots 0 \ddots \\ \hline \ddots 0 \ddots & \ddots 0 \ddots & 0 & \ddots 0 \ddots \\ -\mathbf{g}_j^T & \cdots 0 \cdots & 1 & \cdots 0 \cdots \\ \ddots 0 \ddots & \ddots 0 \ddots & 0 & \ddots 0 \ddots \end{array} \right), \left(\begin{array}{c} d_j\,\mathbf{g}_j \\ \hline 0 \\ -d_j \\ 0 \end{array} \right), d_j^2 \right)$$

$$Q^f = (\mathbf{A}, \mathbf{b}, c) =$$

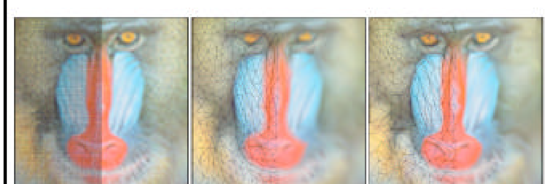
$$\left(\left(\begin{array}{c|c} \mathbf{n}\mathbf{n}^T + \sum_j \mathbf{g}_j\mathbf{g}_j^T & -\mathbf{g}_1 \cdots -\mathbf{g}_m \\ \hline -\mathbf{g}_1^T & \\ \vdots & I \\ -\mathbf{g}_m^T & \end{array} \right), \left(\begin{array}{c} d\,\mathbf{n} + \sum_j d_j\mathbf{g}_j \\ \hline -d_1 \\ \vdots \\ -d_m \end{array} \right), d^2 + \sum_j d_j^2 \right)$$

$$Q = Q_p + \sum_j \lambda_j^2 Q_{s_j},$$

Storage Comparison

Example	m	Previous Q	New Q
geometry	0	10	10
+ color	3	28	23
+ normals	6	55	35
+ texture coord.	8	78	43
in general	$m>0$	$(4+m)(5+m)/2$	$11+4m$

Experiment



(a) Original mesh (79,202 faces) (b) Simplified using Q from [7] (c) Simplified using our new Q

Figure 3: Result of simplifying a vertex-colored 200 × 200 mesh down to 1,000 faces using the previous QEM [7] and using our new QEM. Mesh edges are rendered on the left half of each image. Weights λ relating color to geometric accuracy are set to 1.

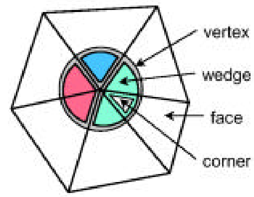
Attribute Discontinuities

Example: a crease ,intensities.

Modeling such discontinuities needs store multiple sets of attribute values per vertex.

Wedges are very useful in this context.

Wedge



Wedge(II)

$$Q^w(\mathbf{v}) = \sum_{f \ni w} \text{area}(f) \cdot Q^f(\mathbf{v}) \ ,$$

$$Q^v(\mathbf{p}, \mathbf{s}^1, \ldots, \mathbf{s}^k) = \sum_{i=1}^{k} Q^{w_i}(\mathbf{p}, \mathbf{s}^i) \ .$$

Wedge unification

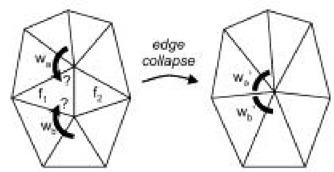


Figure 5: Tests for wedge unification after edge collapse.

Simplification Enhancements

- Memoryless simplification
- Volume preservation

Memoryless simplification

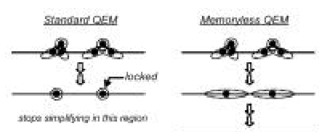


Figure 6: Illustration of standard QEM and memoryless QEM simplification. The dashed ovals symbolize the shapes of the quadric functionals Q_f; (a) in the standard scheme they are are computed once in a preprocess and subsequently summed during simplification; (b) in the memoryless scheme they are computed using the mesh simplified so far.

Volume preservation(I)

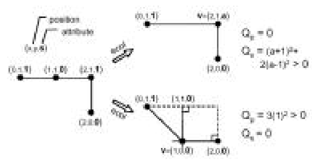


Figure 7: For a polygonal curve in $\mathbf{R}^2$, illustration of trade-off between geometric accuracy ($Q_g = 0$) and attribute accuracy ($Q_a = 0$). Attribute accuracy can cause bias towards the center of curvature when the attribute gradient is high.

Volume preservation(II)

$$\mathbf{g}_{VOL}^T \mathbf{p} + d_{VOL} = 0$$

$$\begin{pmatrix} \mathbf{A} & \mathbf{g}_{VOL} \\ \mathbf{g}_{VOL}^T & 0 \end{pmatrix} \begin{pmatrix} \mathbf{v}_{min} \\ \gamma \end{pmatrix} = \begin{pmatrix} -\mathbf{b} \\ -d_{VOL} \end{pmatrix}.$$

Results(I)

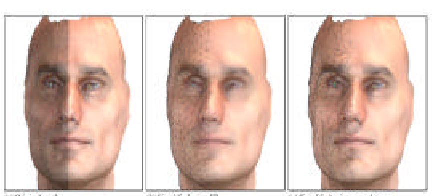
- Distance between two meshes M_1 and M_2 is obtained by sampling a collection of points from M_1 and measuring the distances to the closest points on M_2 plus the distances of the same number of points from M_2 to M_1
- Statistics are reported using L_2 norm and L-infinity norm
- For meshes with attributes, we also sample attributes at the same points and measure the divisions from the values linearly interpolated at the closest point on the other mesh.

Results(II)

Simplification options			rms	max
New Q	Memoryless	$\Delta Vol = 0$	color error	color error
			0.086	0.57
		√	0.087	0.61
	√		0.082	0.57
	√	√	0.082	0.54
√			0.068	0.48
√		√	0.068	0.76
√	√		† 0.071	0.59
√	√	√	0.054	0.33
(time-intensive scheme [10])			0.056	0.33

Table 2: Quantitative accuracy results for 1000-face "mandrill" meshes as in Figure 3, with and without (1) the new QEM, (2) memoryless simplification, and (3) volume preservation. The top row therefore corresponds to the scheme of [7], and the bottom row to our new scheme. The more expensive method from [10] is also included for comparison.

Mesh with color



(a) Original mesh (b) Simplified using [7] (c) Simplified using our scheme

Figure 8: Simplification of a vertex-colored mesh of 135,135 faces down to 1,500 faces.

Results(IV)

Simplification options			Rms error	
New Q	$\Delta Vol = 0$	Memoryless	Geometry	Color
			0.00135	0.035
		√	0.00113	0.035
	√		0.00091	0.029
	√	√	0.00089	0.029
√			0.00167	0.035
√		√	0.00127	0.035
√	√		0.00109	0.027
√	√	√	0.00099	0.027
(time-intensive scheme [10])			0.00095	0.027

Table 3: Results for 1500-face head meshes as in Figure 8.

Results (V)

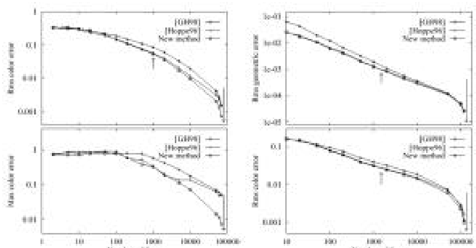
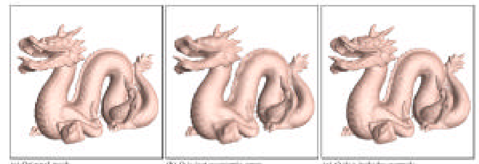


Figure 9: Rms and maximum color error for the "mandrill" model of Figure 3, using [7], [10], and our new scheme.

Figure 10: Accuracy results for the head model of Figure 8, using [7], [10], and our new scheme.

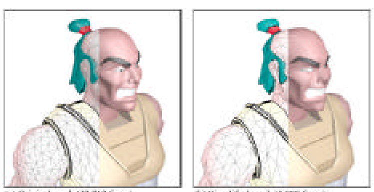
Mesh with normals



(a) Original mesh (b) β is just geometric error (c) β also includes normals

Figure 11: Simplification of a mesh of 928,000 faces down to 10,000 faces. For the geometric simplification in (b), normals are simply carried through. In (c) we optimize both geometry and normals, using λ = 0.02 for normals.

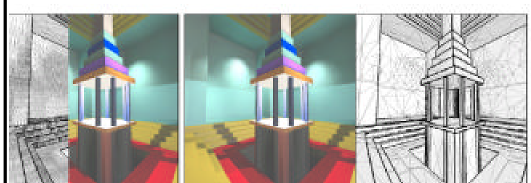
Wedge Attributes



(a) Original mesh (42,712 faces) (b) Simplified mesh (5,000 faces)

Figure 12: Simplification of a mesh with discontinuities on normal attributes (indicated by the thick lines).

Radiosity solution



(a) Original mesh (298,468 faces) (b) Simplified mesh (5,000 faces)

Figure 13: Simplification of a mesh with discontinuities of color attributes.

Results (VI)

Fig.	Rms error								
	Geometry			Color			Normals		
	[10]	[7]	New	[10]	[7]	New	[10]	[7]	New
3	10^{-7}	10^{-7}	10^{-7}	.056	.086	**.054**	-	-	-
8	**.00095**	.00135	.00099	.027	.035	**.027**	.11	.13	**.11**
11c	.00074	.00096	**.00070**	-	-	-	.13	.14	**.11**
12	.00092	.00074	**.00057**	-	-	-	**.26**	.32	.30
13	.00005	.00010	**.00002**	.036	.045	**.034**	.12	.17	**.12**