A Multi-Resolution Technique for Comparing Images Using the Hausdorff Distance

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## Introduction

- Registration methods
  - Correlation and sequential methods
  - Fourier methods
  - Point mapping
  - Elastic model-based matching

## The Hausdorff Distance

- Given two finite point sets $A = \{a_1,\ldots,a_p\}$ and $B = \{b_1,\ldots,b_q\}$

$$H(A, B) = \max(h(A, B), h(B, A))$$

where

$$h(A,B) = \max_{a \in A} \min_{b \in B} \|a - b\|$$

## With transformations

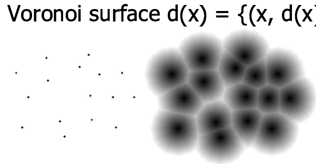
- A: a set of image points
- B: a set of model points
- transformations: translations, scales
  - $t = (t_x, t_y, s_x, s_y)$
  - $w = (w_x, w_y) \Rightarrow t(w) = (s_x w_x + t_x, s_y w_y + t_y)$
  - $f(t) = H(A, t(B))$

## With transformations

- Forward distance
  - $f_B(t) = h(t(B), A)$
  - a hypothesize
- reverse distance
  - $f_A(t) = h(A, t(B))$
  - a test method

## Computing Hausdorff distance

- Voronoi surface $d(x) = \{(x, d(x)) | x \in R^2\}$



$$d(x) = \min_{b \in B} \|x - b\|, \quad d'(x) = \min_{a \in A} \|a - x\|$$

$$H(A,B) = \max(\max_{a \in A} d(a), \max_{b \in B} d'(b))$$

## Computing Hausdorff distance

- The forward distance for a transformation $t = (t_x, t_y, s_x, s_y)$

$$f_B(t) = h(t(B), A) = \max_{b \in B} \min_{a \in A} \| a - t(b) \|$$
$$= \max_{b \in B} \min_{a \in A} \| a - (s_x b_x + t_x, s_y b_y + t_y) \|$$
$$= \max_{b \in B} d'(s_x b_x + t_x, s_y b_y + t_y)$$

## Comparing Portion of Shapes

- Partial distance

$$h_K(t(B), A) = \underset{b \in t(B)}{K^{th}} \min_{a \in A} \| a - b \|$$

- The partial bidirectional Hausdorff distance
  - $H_{LK}(A,t(B))=\max(h_L(A,t(B)), h_K(t(B),A))$

## A Multi-Resolution Approach

- Claim 1.

  $0 \le b_x \le x_{max}$, $0 \le b_y \le y_{max}$ for all $b=(b_x,b_y) \in B$,

  $t = (t_x, t_y, s_x, s_y)$,

$$\rho(t,t') = \| (|s_x - s_x'| x_{\max} + |t_x - t_x'|, |s_y - s_y'| y_{\max} + |t_y - t_y'|) \|$$

$$\Rightarrow \quad |\delta - \delta'| \le \rho(t,t')$$

## A Multi-Resolution Approach

- If $H_{LK}(A, t(B)) = \nu > \tau$, then any transformation $t'$ with $\rho(t,t') < \nu - \tau$ cannot have $H_{LK}(A, t'(B)) \le \tau$
- A rectilinear region(cell)

$$R = [t_x^{low}, t_x^{high}] \times [t_y^{low}, t_y^{high}] \times [s_x^{low}, s_x^{high}] \times [s_y^{low}, s_y^{high}]$$

- The center of this cell

$$t_c = ((t_x^{low} + t_x^{high})/2, (t_y^{low} + t_y^{high})/2, (s_x^{low} + s_x^{high})/2, (s_y^{low} + s_y^{high})/2)$$

- If $\delta = H_{LK}(A, t_c(B))$

$$> \tau + \| (x_{\max}(s_x^{high} - s_x^{low})/2 + (t_x^{high} - t_x^{low}), y_{\max}(s_y^{high} - s_y^{low})/2 + (t_y^{high} - t_y^{low})) \|$$

  then no transformation $t' \in R$ have $H_{LK}(A, t'(B)) \le \tau$

## A Multi-Resolution Approach

1. Start with a rectilinear region(*cell*) which contains all transformations
2. For each cell,

   If $H_{LK}(A, t_c(B))$

   $> \tau + \| (x_{\max}(s_x^{high} - s_x^{low})/2 + (t_x^{high} - t_x^{low}), y_{\max}(s_y^{high} - s_y^{low})/2 + (t_y^{high} - t_y^{low})) \|$

   Mark this cell as *interesting*
3. Make a new list of cells that cover all interesting cells.
4. Repeat 2,3 until the cell size becomes small enough

## The Hausdorff distance for grid points

- $A = \{a_1, \ldots, a_p\}$ and $B = \{b_1, \ldots, b_q\}$ where each point $a \in A$, $b \in B$ has integer coordinates
- The set A is represented by a binary array A[k,l] where the k,l-th entry is nonzero when the point (k,l) $\in$ A
- The distance transform of the image set A
  - D′[x,y] is zero when A[k,l] is nonzero,
  - Other locations of D′[x,y] specify the distance to the nearest nonzero point

## Rasterizing transformation space

- Translations
  - An accuracy of one pixel
- Scales
  - $b=(b_x,b_y) \in B$, $0 \le b_x \le x_{max}$, $0 \le b_y \le y_{max}$
  - x-scale: an accuracy of $1/x_{max}$
  - y-scale: an accuracy of $1/y_{max}$
  - Lower limits: $s_{xmin}$ , $s_{ymin}$
  - Ratio limits: $s_x/s_y > a_{max}$ or $s_y/s_x > a_{max}$
- Transformations can be represented by
  - $(i_x, i_y, j_x, j_y)$ represents the transformation $(i_x, i_y, j_x/x_{max}, j_y/y_{max})$

## Rasterizing transformation space

- Restrictions where A[k,l], $0 \le k \le m_a$, $0 \le l \le n_a$

$$s_x^{\min} x_{\max} \le j_x \le x_{\max}$$
$$s_y^{\min} y_{\max} \le j_y \le y_{\max}$$
$$\frac{y_{\max}}{x_{\max} a_{\max}} \le \frac{j_x}{j_y} \le \frac{y_{\max} a_{\max}}{x_{\max}}$$
$$0 \le i_x \le m_a - j_x$$
$$0 \le i_y \le n_a - j_y$$

## Rasterizing transformation space

- Forward distance $h_K(t(B),A)$

$$F_B[i_x, i_y, j_x, j_y] = \underset{b \in B}{K^{th}} D'[< j_x b_x / x_{\max} + i_x >, < j_y b_y / y_{\max} + i_y >] \cdot$$

- Bidirectional distance

$$F[i_x, i_y, j_x, j_y] = \max(F_A[i_x, i_y, j_x, j_y], F_B[i_x, i_y, j_x, j_y])$$

## Reverse distances in cluttered images

- When the model is considerably smaller than the image
  - Compute a partial reverse distance only for image points near the current position of the model

$$F_A[i_x, i_y, j_x, j_y] = \underset{\substack{(k,l) \\ i_x \le k < i_x + j_x \\ i_y \le l < i_y + j_y}}{L^{th}} A[k,l] D'[k,l]$$

## Increasing the efficiency of the computation

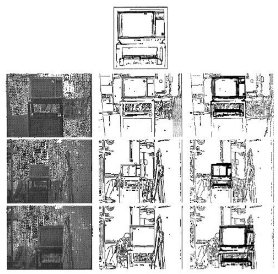
- Early rejection
  - $K = \lfloor f_1 q \rfloor$ where $q = |B|$
  - If the number of probe values greater than the threshold exceeds q-K, then stop for the current cell
- Early Acceptance
  - Accept "false positive" and no "false negative"
  - A fraction s, $0<s<1$ of the points of B
  - When s=.2
    - 10% of cells labeled as interesting actually are shown to be uninteresting

## Increasing the efficiency of the computation

- Skipping forward
  - Relies on the order of cell scanning
  - Assume that cells are scanned in the x-scale order.
  - $D'_{+x}[x,y]$: the distance in the increasing x direction to the nearest location where $D'[x,y] \le \tau'$
  - Computing $D'_{+x}[x,y]$: Scan right-to-left along each row of $D'[x,y]$
  - $\gamma_x = \frac{x_{\max}}{b_x} \max(0, D'_{+x}[< j_x b_x / x_{\max} + i_x >, < j_y b_y / y_{\max} + i_y >] - 1)$
  - $F_B[i_x,i_y,j_x,j_y],\ldots,FB[i_x,i_y,j_x+\gamma-1,j_y]$ must be greater than $\tau'$

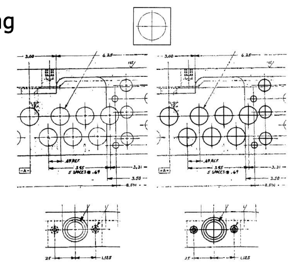
## Examples

- Camera images
  - Edge detection



## Examples

- Engineering drawing
  - Find circles



## Conclusion

- A multi-resolution method for searching possible transformations of a model with respect to an image
- Problem domain
  - Two dimensional images which are taken of an object in the 3-dimensional world
  - Engineering drawings